PORTFOLIO & RISK
ANALYTICS RESEARCH

A Bloomberg Professional Service Offering

PORTFOLIO VALUE-AT-RISK

Bloomberg

CONTENTS

PORTFOLIO & RISK ANALYTICS OVERVIEW

Bloomberg's Portfolio and Risk Analytics solution, available via the Bloomberg Professional® service, offers a comprehensive set of customizable tools for the desktop. In today's market, your company's performance depends on the ability to understand and manage risk and consistently out-think the market. Bloomberg supports portfolio managers, risk managers and senior management with a series of new and enhanced tools to systematically analyze and track portfolio risk, and to construct and rebalance portfolios that optimally achieve investment objectives and criteria.

PORTFOLIO VALUE-AT-RISK

The new VAR tab is a part of Bloomberg's suite of portfolio and risk analytics provided by PORT <GO>. It enables risk managers and portfolio managers to analyze the tail risk of their portfolios using the latest risk modeling techniques.

This document describes the new methodology for portfolio value-at-risk (VaR) computation provided by Bloomberg Portfolio and Risk Analytics. Three types of VaR are provided:

1. Parametric VaR
2. Historical VaR
3. Monte Carlo VaR

The new VaR methodology utilizes the factor structure provided by the Bloomberg factor models, in a way that makes the VaR consistent with portfolio tracking error and volatility that are computed using the same factor models. For Historical and Monte Carlo VaR an array of valuation choices are offered, ranging from linear pricing using the Bloomberg factor models to Stress Matrix Pricing (SMP) and full valuation. In the remainder of this document we describe the different components of Bloomberg VaR calculation, namely, the Bloomberg factor models, security valuation methods, and details of Parametric, Historical and Monte Carlo VaR calculation.

BLOOMBERG FUNDAMENTAL FACTOR MODELS

Reliable estimation of portfolio volatility is a key first step towards computing reliable VaR estimates. Bloomberg uses linear factor models to estimate portfolio volatility.

Factor models have become an indispensable tool for modern portfolio management as well as risk management. They provide greater understanding of sources of portfolio risk, and the ability to attribute portfolio performance, to forecast both absolute risk and benchmark-relative risk and to improve portfolio construction. Recent market volatility highlights the importance of controlling unwanted factor exposures in portfolios. While factor models have been in use for at least two decades, the quantitative equity hedge fund meltdown of August 2007, the market collapse in the wake of the Lehman Brothers bankruptcy, and extreme volatility of several factors since then have attracted the attention of traditional and quantitative portfolio managers alike and have dramatically increased client interest in factor models.

Factor models are based on the basic principle that security returns are driven by a set of common factors. Therefore, portfolio risk depends on volatility and correlation of these factors and on the amount of portfolio exposure to individual factors. Additionally, there are risks not captured by the common factors; factor models help estimate these "non-factor" risks as well.

Bloomberg's approach to constructing risk factor models uses a combination of explicit and implicit factors. An implicit or fundamental factor model is constructed by defining security exposures to each factor and then imputing factor returns from a regression of security returns on the exposures. While this class of models has several advantages over the alternatives, we chose this approach primarily due to its better interpretability by the user. It gives greater insight into the portfolio risk sources and leads to intuitive action items. Additionally, explicit factors are used when the impact of certain observable factors on security returns is known. For example, FX rates are used as explicit factors in equity factor models, and changes in the curve are used in fixed income factor models. Factor exposures for explicit factors are analytically computed. The single-period return of the nth security in the tth time period is modeled by Bloomberg factor models as

$$r_{nt} = \sum_{k=1}^{K} X_{nkt} f_{kt} + \epsilon_{nt},$$

where X_{nkt} is the exposure of the nth security to the kth factor at time t f_{kt} is the kth factor return at time t, K is the number of factors, and ϵ_{nt} is the non-factor return of the nth security at time t.

The model assumes that the factor returns are uncorrelated with the non-factor returns and that the non-factor returns have sparse correlations. For example, for equities we assume that non-factor returns are mutually uncorrelated except in cases of multiple share classes of the same company and ADR/GDRs.

For certain credit instruments we assume that non-factor returns corresponding to different issuers are mutually uncorrelated. The number of factors and their definition depend on the particular factor model that is used. Bloomberg offers several fundamental factor models based on the asset class and region of asset coverage. Please see the model white papers for an in-depth description of individual factor models.

The factor model shown on the previous page for security returns may be written in matrix notation as

$$R_t = X_t F_t + \epsilon_t,$$

where R_t is the vector of N security excess returns at time t, X_t is the $N \times K$ matrix of factor exposures, F_t is the vector of K factor returns at time t ϵ_t is the vector of N non-factor returns at time t.

From this model we can derive the asset return covariance matrix as

$$Q_t = X_t \Sigma_t X_t' + D_t,$$

where t is the factor return covariance matrix, and D_t is the sparse matrix of non-specific returns.

The implicit factor returns at time t are estimated using weighted cross-sectional regression of the asset returns on factor exposures at time t. The estimated factor returns are then used to estimate the factor covariance matrix Σ_t, using exponential averaging and shrinkage. The choice of half-life for exponential averaging for factor variances is based on the specific factor model—please see documentation on the individual Bloomberg factor models. For example, Bloomberg uses a half-life of 26 weeks for exponential averaging of variances and 52 weeks for correlations for equity models.

The volatility of a portfolio with weights specified by the vector w can be computed using the above factor model as

$$\sigma_{wt} = \sqrt{w' Q_t w} = \sqrt{w' X_t \Sigma_t X_t' w + w' D_t w}.$$

Bloomberg factor models are used both for valuation and risk modeling as part of the VaR methodology, as explained in the following sections.

VALUATION METHODOLOGY

An integral part of VaR calculation is the valuation of each security in the portfolio and aggregation of returns across the portfolio to construct the entire return distribution. One of the four valuation methods described in the following is selected for each security based on the type of the security and user preferences. The choice of valuation method for a given security aims to achieve computational efficiency without sacrificing accuracy.

Linear Factor Model Pricing

The Bloomberg factor models described previously are used by this method to compute the return of a security, given factor returns and the non-factor return. This method is implicitly used for all securities in computing Parametric VaR, as explained in Section 4. This method is also used for Historical and Monte Carlo VaR for securities whose price is accurately modeled as a linear combination of (explicit or implicit) model factors. Examples of such securities are equities and fixed income securities without strong convexity or optionality.

Each scenario in the computation of Historical and Monte Carlo VaR specifies a set of factor returns and non-factor returns, which are fed into the factor model to compute the corresponding security returns.

For securities with optionality, the Delta/Gamma or Duration/Convexity approximation is used, which approximates the pricing function by the first-and-second-order terms in its Taylor expansion. Note that the presence of the second order (gamma or convexity) term makes this a non-linear approximation. But this non-linearity is converted to a linear operation on factors by defining market-wide factors that approximate the per-security non-linear terms. Consider the example of a vanilla equity option: the gamma term includes the squared return of the underlying asset, which we approximate by the "market-square" factor (see documentation of the Bloomberg factor models). The analogous factor in the case of fixed income securities is the convexity factor. This approximation linearizes the pricing function with respect to the factor returns and makes it suitable for Parametric VaR computation.

Delta/Gamma Pricing

Delta/Gamma or Duration/Convexity pricing is an available pricing method for securities with optionality in Historical and Monte Carlo VaR computation. This is similar to the Delta/Gamma approximation described for linear factor model pricing with one exception: When computing VaR using historical to Monte Carlo simulations we have access to all the underlying factors for each scenario, which enables us to model the true non-linearity of the gamma term instead of using market-wide approximations. For example, in the case of equity options this method uses the squared return of the underlying stock for each scenario instead of the market square factor approximation. Exact modeling of the gamma term makes this pricing method more accurate than linear pricing for simulation-based VaR computation. However, since it ignores higher-order Greeks, this method may not be suitable for options with significant non-linearities in the pricing function.

Stress Matrix Pricing

The Stress Matrix Pricing (SMP) approach offers a compromise between the accuracy we achieve using Full Valuation and the speed of Delta/Gamma pricing. It is available in Historical and Monte Carlo VaR computation for derivative securities, which include equity options and interest rate and credit products with embedded options.

The SMP approach to pricing is as follows. To avoid the computational effort required to fully value every scenario, we first store the difference between the true price computed using full valuation and the Delta/Gamma price for a much smaller set of scenarios (the "stress matrix"). We then compute the Delta/Gamma price for each scenario in a simulation set and apply a non linearity correction that is interpolated from the stress matrix. This approach gives us the exact price if the scenario exactly matches one of the stored scenarios, but may result in an interpolation error for other scenarios.

For a detailed explanation of the SMP methodology, please see Bloomberg's Stress Matrix pricing white paper.

Full Valuation

For some securities with highly non-linear pricing functions, such as certain exotic derivatives and short time-to-maturity options, we find that the Delta/Gamma Pricing and SMP approaches do not accurately capture the true distribution of security returns, given a distribution of the underlying risk factors. In such cases we offer the flexibility to use full pricing of the security in all scenarios, when Historical or Monte Carlo VaR is selected.

While this method provides the most accurate return distribution given a distribution of factors, it is also the most computationally expensive, and will be used only when the cost in accuracy is too great for the other methods.

VAR COMPUTATION

Bloomberg offers the following three choices for VaR computation. Table 1 on the next page summarizes the parameters and techniques used in computing the different VaR estimates.

Parametric VaR

The Parametric VaR methodology follows the traditional approach of assuming a jointly normal distribution among all assets in the portfolio to compute a VaR estimate analytically. It makes use of the Bloomberg factor models that provide the asset covariance matrix in terms of the factor covariance matrix, factor exposures and non-factor variances, and models the factor returns and non-factor returns as jointly normal random variables. These assumptions, coupled with the use of linear pricing (see Section 3), imply a normal distribution for the portfolio return. The standard deviation of the normally distributed portfolio return is the portfolio volatility, which is computed as shown in Section 2.

The advantages of the parametric approach to computing VaR are very high speed of computation and compatibility with traditional reporting systems that include this estimate.

However, as is increasingly recognized by risk practitioners, realized distributions of portfolio returns are significantly non-normal—they exhibit fat-tailed behavior, which means that extreme moves in portfolio return occur with a much larger probability than that predicted by a normal distribution. Therefore, Parametric VaR tends to underestimate VaR at very high confidence levels. Parametric VaR also imposes the restriction of linear pricing, which is not suitable for highly non-linear securities. Historical and Monte Carlo VaR estimates described below aim to overcome these drawbacks of Parametric VaR at the expense of a higher computational cost.

Historical VaR

This VaR methodology models fat-tailed behavior of returns by using the distribution of realized (historical) factor returns instead of making the assumption that factor returns are normally distributed. The joint distribution of factor returns is represented by a panel of historical daily returns over multiple years of recorded factor history, which we call historical simulations. We simulate the corresponding non-factor returns by drawing from (fat-tailed) Student's t distributions whose standard deviations are the current estimates of security non-factor volatilities.

We use the factor returns and non-factor returns for each historical scenario and the current factor exposures to compute the corresponding returns of all securities using one of the pricing methods described in Section 3, and aggregate the returns across the portfolio. This yields a historical sequence of portfolio returns given the current portfolio holdings. Historical VaR is then computed as the desired percentile of the portfolio return distribution, e.g., the 5th percentile portfolio return represents the Historical VaR at the 95% confidence level. Since the return distribution is given by historical returns, the choice of the length of the historical period is a critical input for Historical VaR.

In contrast to Parametric VaR, Historical VaR captures the fat-tailed behavior of portfolio returns and offers the flexibility of using multiple valuation techniques described in Section 3. The main advantage of Historical VaR over Monte Carlo VaR is the fact that it makes no assumptions on the joint return distribution other than that the future return distribution is the same as the historical distribution. This often makes Historical VaR easier to interpret and explain. On the other hand, one may question the validity of using the historical distribution for the distribution of future returns, since current market conditions may be quite different from those experienced in the past. Historical VaR is also limited by the length of historical data chosen for historical simulations; a small number of historical scenarios would lead to a lower statistical confidence in the VaR estimate.

	PARAMETRIC	HISTORICAL	MONTE CARLO
Number of Simulations	N/A	Historical 1 year: 250 Historical 2 year: 500 Historical 3 year: 750	10,000
Distributional Assumption	Normal	Empirical	Student's t Marginal
Time Series Weights	Exponentially Weighted	Equally Weighted	Exponentially Weighted
Half-Life	26 Weeks for Volatility and 52 weeks for Correlations	None	26 Weeks for Volatility and 52 Weeks for Correlations
Time Horizon Projection	Daily VaR is Scaled by the Square Root of the Horizon	Daily VaR is Scaled by the Square Root of the Horizon	Daily VaR is Scaled by the Square Root of the Horizon
Simulated Non-factor Risk	N/A	Non-factor Volatility From the Multi-factor Risk Models Are Used as the Basis to Simulate Non-factor Returns (Student's t Assumption for Non-factor Risk)	Non-factor Volatility From the Multi-factor Risk Models Are Used as the Basis to Simulate Non-factor Returns (Student's t Assumption of Non-factor Risk)

Table 1: Summary of VaR methodologies

Monte Carlo VaR

The Monte Carlo approach to VaR estimation is to estimate the joint distribution of future factor and non-factor returns and to draw a large number of random simulations from this joint distribution to create Monte Carlo scenarios. This enables us to use a forward-looking distribution of the market rather than a backward-looking distribution that historical simulations represent. It also increases the statistical accuracy of VaR estimation compared with Historical VaR, due to the use of a very large number of scenarios.

In order to provide us with added flexibility when formulating the multivariate distribution required in generating VaR, we separate the modeling of the marginal distribution of each risk factor from that of the dependence structure across factors. Bloomberg uses fat-tailed marginal distributions to model the distribution of each individual factor return, and a fat-tailed copula to model the inter dependence of factors, thus going beyond the assumption of jointly normal factor returns. The marginal distributions of individual factors are modeled from the family of Student's t distributions. The degrees of freedom parameter of the t distribution, which determines the thickness of its tails, is calibrated to historical factor return data for each factor. Bloomberg uses a normal or a Student's t copula to model the dependence structure between factors.

One of the advantages of using copulas is that they isolate the dependence structure from the structure of the marginal distributions. This separation allows us flexibility in independently choosing the most appropriate models for the marginal distributions of individual factors and the copula for their interdependence. Bloomberg's implementation of Monte Carlo VaR follows the steps shown above.

1. Estimate the marginal distributions of individual factors: To capture the fat-tailed behavior of the market we use the Student's t distribution with appropriate degrees of freedom as a parametric model for the marginal distribution of each risk factor. The family of t distributions generalizes the normal distribution, and includes the normal distribution as a special case. We fit a separate t distribution to each factor in the risk model. The degrees of freedom of the Student's t-distribution, which determines the fatness of its tails, is estimated using the maximum-likelihood method, and the variance is estimated using exponentially weighted moving averaging (EWMA) on historical factor data. See documentation of individual Bloomberg factor models for details on the estimation of factor covariance matrix.

2. Estimate the factor copula: We model the inter dependence structure of the joint distribution of factors using a parametric copula distribution. Bloomberg currently uses a t copula with six degrees of freedom, and will provide a choice to the user in the future to select from a normal or a t copula. The copula is parameterized by its correlation matrix, which we estimate using the historical risk factor data. We use exponential weighting with a half-life of 52 weeks and shrinkage to estimate the copula correlation matrix. The copula correlation matrix provides the basis for the scenario generation of joint returns of multiple risk factors.
3. Generate Monte Carlo simulations: Bloomberg draws 10,000 random simulations of factor returns and non-factor returns from the joint distribution estimated as described above. This is done in two steps: We first sample from the copula distribution to obtain a set of 10,000 scenarios that determine the inter dependence structure of factor and non-factor returns. We then transform these scenarios to a panel of joint factor and non-factor returns using the marginal distributions estimated in Step 1.

Once the panel of Monte Carlo scenarios is generated in this manner, the computation of Monte Carlo VaR follows the same method as Historical VaR: each security is priced using the most appropriate valuation method from Section C, security returns are aggregated to form scenarios of portfolio return using the current portfolio holdings, and Monte Carlo VaR is computed as the desired percentile of the Monte Carlo distribution of portfolio return.

DUBAI
+971 4 364 1000

HONG KONG
+852 2977 6000

MILAN
+39 02 8064 4225

NEW YORK
+1 212 318 2000

SÃO PAULO
+55 11 2395 9000

SYDNEY
+61 2 9777 8600

FRANKFURT
+49 69 9204 1210

LONDON
+44 20 7330 7500

MUMBAI
+91 22 6120 3600

SAN FRANCISCO
+1 415 912 2960

SINGAPORE
+65 6212 1000

TOKYO
+81 3 3201 8900

bloomberg.com